Defeating with Fault Injection a Combined Attack Resistant Exponentiation

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Agenda

1. What is a combined attack?
2. Algorithm of Schmidt et al.
3. Fault attack
   1. On simplified algorithm
   2. On complete algorithm
4. Combined attack
5. Improved algorithm
6. Conclusion

What is a combined attack?

General principle

- Combines a fault attack with a leakage analysis
- Main goal: attack implementations resistant against fault and leakage analysis
- New implementations and new countermeasures often required

What is a combined attack?

Example on L2R exponentiation

Algorithm 1 Left-to-right multiply always exponentiation

Input: $x \in \mathbb{G}$ and $d = (d_{k-1}, \ldots, d_0)_2 \in \mathbb{N}$

Output: x^d

$R[0] \leftarrow 1$

$R[1] \leftarrow x$

$t \leftarrow 0$

for $i = k - 1$ to 0 **do**

  $R[0] \leftarrow R[0].R[t]$

  $t \leftarrow t \oplus d_i$

  $i \leftarrow i - 1 + t$

end for

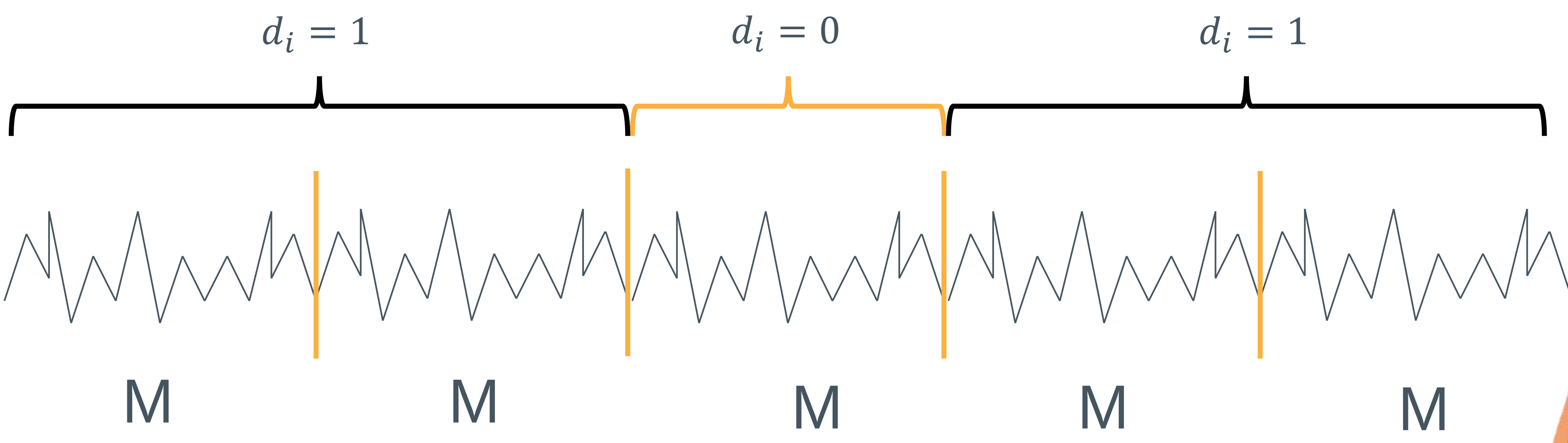
return $R[0]$

<u>Add:</u> classical fault checking mechanism

- *inverse operation calculation* or
- *doubling the calculation to verify equality of both*

What is a combined attack?

Example on L2R exponentiation



$$d = (101)_2$$

No SPA leakage, only multiplications

What is a combined attack?

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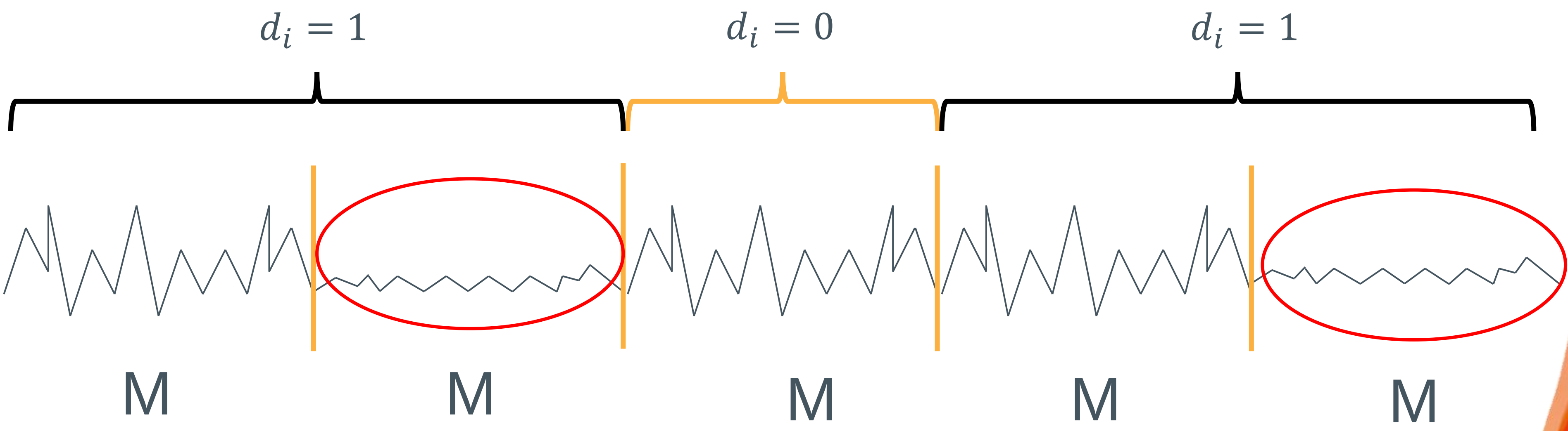
return $R[0]$

Skip instruction

Suppose $R[1] = 0$

What is a combined attack?

Example on L2R exponentiation



$$d = (101)_2$$

The use of the faulted register $R[1]$ is visible by SPA

What is a combined attack?

History on asymmetric

- **2007: Attack on atomic left-to-right exponentiation**
  - *Amiel et al. (FDTC)*
- **2010: Resistant algorithms for RSA and ECC**
  - *Schmidt et al. (LATINCRYPT)*
- **2011: Attack on scalar multiplication**
  - *Fan et al. (CHES)*
- **2012: Attack on prime generation**
  - *Vuillaume et al. (COSADE)*
- **2013: Attack on RSA-CRT**
  - *Barbu et al. (PKC)*

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Algorithm of Schmidt et al.

General principle

- Add to (*potentially any*) SPA-resistant exponentiation
  - An infective computation method
  - An invariant system
- Link those two protections to strengthen the resistance

Algorithm of Schmidt et al.

Exponentiation

Algorithm 1 Schmidt *et al.* [20, Alg. 3] left-to-right exponentiation.

Input: $d = (d_{t-1}, \ldots, d_0)_2, m \in \mathbb{Z}_N, N$ and block length W.
Output: $m^d \bmod N$

$r_1 \leftarrow \mathsf{random}(1, 2^\lambda - 1)$
$r_2 \leftarrow \mathsf{random}(1, 2^\lambda - 1)$
$i \leftarrow (r_2^{-1} \bmod N) \cdot r_2$
$R_0 \leftarrow i \cdot 1 \bmod N r_2$
$R_1 \leftarrow i \cdot m \bmod N r_2$
$\bar{d} \leftarrow d + r_1 \cdot \varphi(N)$
$[\tilde{d}^{(l-1)}, \ldots, \tilde{d}^{(0)}] \leftarrow [\psi_0(\bar{d}^{(l-1)}), \ldots, \psi_0(\bar{d}^{(0)})]$
$k \leftarrow 0$
$j \leftarrow \mathsf{bitlength}(\tilde{d}) - 1$
while $j \geq 0$ **do**
    $R_0 \leftarrow R_0 \cdot R_k \bmod N r_2$
    if $(R_0 = 0)$ or $(R_1 = 0)$ **then**
        $[\tilde{d}^{(l-1)}, \ldots, \tilde{d}^{(0)}] \leftarrow [1, \ldots, 1]$
    end if
    $\hat{d} \leftarrow \psi^{-1}_{(R_0 + R_1 \bmod r_2)}(\tilde{d}^{(\lfloor j/W \rfloor)})$
    $k \leftarrow k \oplus \mathsf{bit}(\hat{d}, j \bmod W)$
    $j \leftarrow j - \neg k$
end while
$c \leftarrow R_0 \bmod N$
return c

Algorithm of Schmidt et al.

Invariant

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Idempotent element $i \in \mathbb{Z}_{N r_2}$ such that:

- $i \equiv 1 \bmod N$
- $i \equiv 0 \bmod r_2$

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i "mixed in" the registers R_0 and R_1

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Efficient test of integrity:

$R_0 \bmod r_2 \mathrel{?=} 0 \bmod r_2$
$R_1 \bmod r_2 \mathrel{?=} 0 \bmod r_2$

Algorithm of Schmidt et al.

Infective computation

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Encode the exponent using
$\psi_\alpha : \mathbb{Z}_{r_2} \times \mathbb{Z}_{r_2} \rightarrow \mathbb{Z}_{r_2}$

- $\psi_\alpha(d^{(j)}) = (\alpha + N)^{-1} . d^{(j)} \bmod r_2$
- $\psi_\alpha^{-1}(\tilde{d}^{(j)}) = (\alpha + N) . \tilde{d}^{(j)} \bmod r_2$

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If $\alpha = 0 \bmod r_2$
Correct decoding
Else
False decoding

Algorithm of Schmidt et al.

Link invariant and infective computation

Algorithm 1 Schmidt *et al.* [20, Alg. 3] left-to-right exponentiation.

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If $\alpha = 0 \bmod r_2$

Correct decoding

Else

False decoding

$\alpha := \mathrm{R}_0 + \mathrm{R}_1 \bmod \mathrm{r}_2$ is the invariant check

Algorithm of Schmidt et al.

Additional check

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$c \leftarrow R_0 \bmod N$
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If R_0 or R_1 is erased by fault
Corrupt the exponent

Check against the combined attack of Amiel et al. 2007

Output

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Returns (*possibly*) faulted results ☹

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Fault attack

On simplified algorithm

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Simplified version: no exponent blinding

Fault attack

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```
r_1 ← random(1, 2^λ − 1)
r_2 ← random(1, 2^λ − 1)
i ← (r_2^{−1} mod N) · r_2
R_0 ← i · 1 mod N r_2
R_1 ← i · m mod N r_2
d̄ ← d + r_1 · φ(N)
[d̃^{(l−1)}, ..., d̃^{(0)}] ← [ψ_0(d̄^{(l−1)}), ..., ψ_0(d̄^{(0)})]
k ← 0
j ← bitlength(d̃) − 1
while j ≥ 0 do
    R_0 ← R_0 · R_k mod N r_2
    if (R_0 = 0) or (R_1 = 0) then
        [d̃^{(l−1)}, ..., d̃^{(0)}] ← [1, ..., 1]
    end if
    d̂ ← ψ^{−1}_{(R_0 + R_1 mod r_2)}(d̃^{(⌊j/W⌋)})
    k ← k ⊕ bit(d̂, j mod W)
    j ← j − ¬k
end while
c ← R_0 mod N
return c
```

Simplified version: no exponent blinding

Skip instruction

$\hat{d} = \psi_0^{-1}(1) = N \bmod r_2$

For the rest of the exponentiation!

Fault attack

On simplified algorithm

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For the rest of the exponentiation!

Only W bits of $\hat{d}$ are used

Let $H = (N \bmod r_2) \bmod 2^W$

Fault attack

On simplified algorithm

- **Attacker knows v first bits of the exponent**
- **Fault u bits after in the loop**
- **Faulted exponent $\breve{d}_u$ of the result $\breve{S}_u$:**

$$\breve{d}_u = \underbrace{\sum_{i=\tilde{t}-v}^{\tilde{t}-1} 2^i.\tilde{d}_i}_{\text{Known part}} + \underbrace{\sum_{i=\tilde{t}-v-u}^{\tilde{t}-v-1} 2^i.\tilde{d}_i}_{u \text{ bits to retrieve}} + \sum_{i=0}^{\tilde{t}-v-u-1} 2^i.H_{(i \bmod W)}$$

Known part u bits to retrieve

with $\tilde{t}$ the bit size of the encoded exponent $\tilde{d}$

Fault attack

On simplified algorithm

- Faulted exponent $\breve{d}_u$:

$$\breve{d}_u = \underbrace{\sum_{i=\tilde{t}-v}^{\tilde{t}-1} 2^i . \tilde{d}_i}_{\text{Known part}} + \underbrace{\sum_{i=\tilde{t}-v-u}^{\tilde{t}-v-1} 2^i . \tilde{d}_i}_{u \text{ bits to retrieve}} + \sum_{i=0}^{\tilde{t}-v-u-1} 2^i . H_{(i \bmod W)}$$

Known part u bits to retrieve

- Guesses: u bits of d and W bits of H
- → Complete guessed result $S_g(u, H)$
- Validate guess by checking: $S_g(u, H) ?= \breve{S}_u$

Fault attack

On simplified algorithm

- **Attack retrieves u bits at a time**
- **Only possible if no exponent blinding**

- **Computational complexity:**

$$\mathcal{C} = \mathcal{O}\left(\frac{2^{(u+W)}.\,\tilde{t}}{u}\right)$$

- **Number of faults:**

$$\mathcal{F} = \mathcal{O}\left(\frac{\tilde{t}}{u}\right)$$

Fault attack

On simplified algorithm

- **Example of computational complexities for $u = 1$**

W	512 bits	1024 bits	2048 bits
8	$\mathcal{C} = 2^{18}$	$\mathcal{C} = 2^{19}$	$\mathcal{C} = 2^{20}$
16	$\mathcal{C} = 2^{26}$	$\mathcal{C} = 2^{27}$	$\mathcal{C} = 2^{28}$
32	$\mathcal{C} = 2^{42}$	$\mathcal{C} = 2^{43}$	$\mathcal{C} = 2^{44}$

- **Validated on PC using the GMP library**

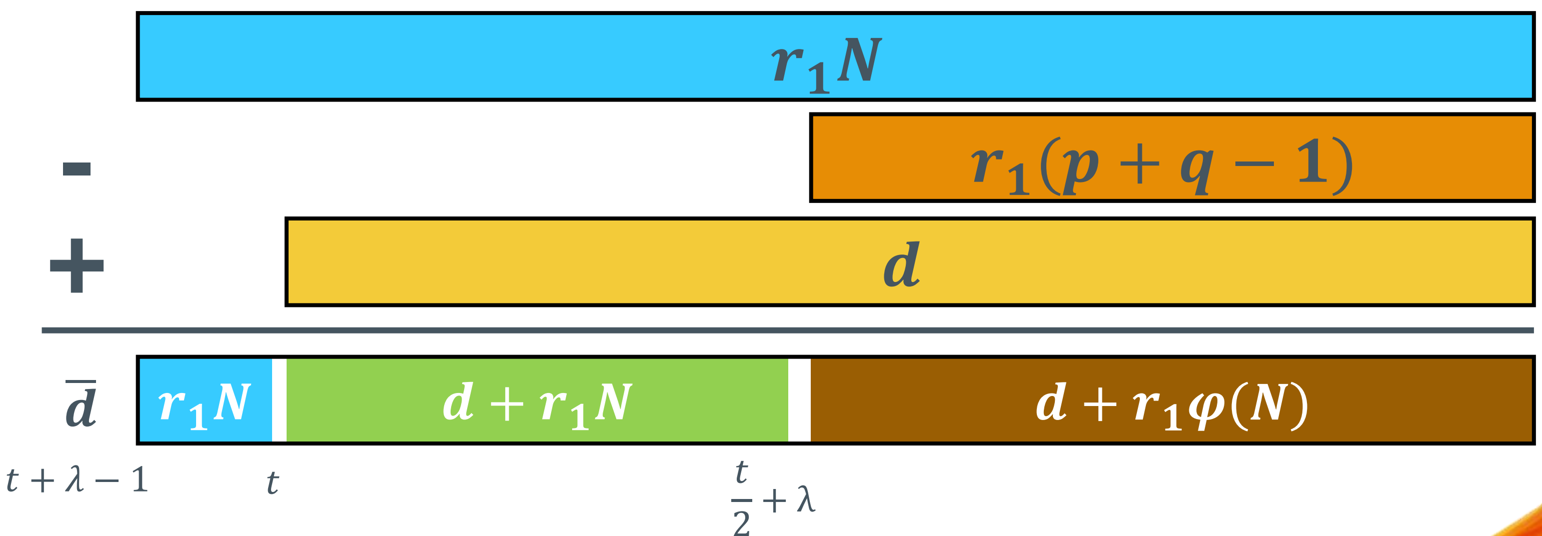
Agenda

Fault attack

On complete algorithm

- **Exponent blinding:** $\bar{d} = d + r_1\varphi(N) = d + r_1 N - r_1(p+q-1)$
- **Effect of the blinding:**

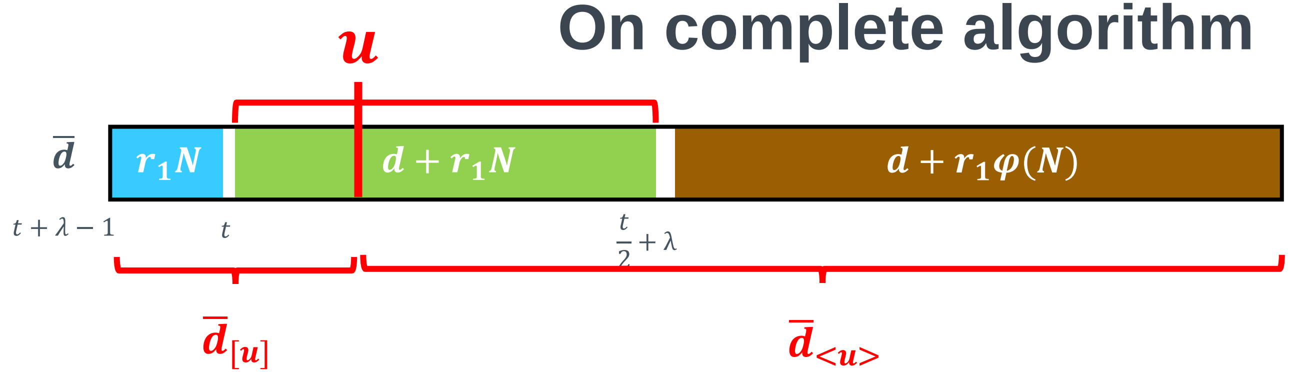
$$\bar{d} = \sum_{i=t}^{t+\lambda-1} 2^i.(r_1N)_i + \sum_{i=\frac{t}{2}+\lambda}^{t-1} 2^i.(d+r_1N)_i + \sum_{i=0}^{\frac{t}{2}+\lambda-1} 2^i.\left(d+r_1\varphi(N)\right)_i$$



Source: Berzati et al. CHES 2010

Fault attack

On complete algorithm



- <u>MSB part of d</u>
- Let $u \in \left[\frac{t}{2} + \lambda, t\right]$
- Let $\overline{d}_{[u]} = \sum_{i=t-u}^{t+\lambda-1} 2^i . \overline{d}_i$ and $\overline{d}_{<u>} = \sum_{i=0}^{t-u-1} 2^i . \overline{d}_i$

  $\overline{d}_i \rightarrow H_{(i \bmod W)}$

- Approximation of $\overline{d}_{[u]}$:

$$\overline{d}_{[u]} \approx \sum_{i=t-u}^{t+\lambda-1} 2^i . (d + r_1 N)_i$$

$$\approx d_{\text{known}} + \sum_{i=t-v-u}^{t-v-1} 2^i . d_i + \sum_{i=t-v-u}^{t+\lambda-1} 2^i . (r_1 N)_i + \text{carry}$$

Fault attack

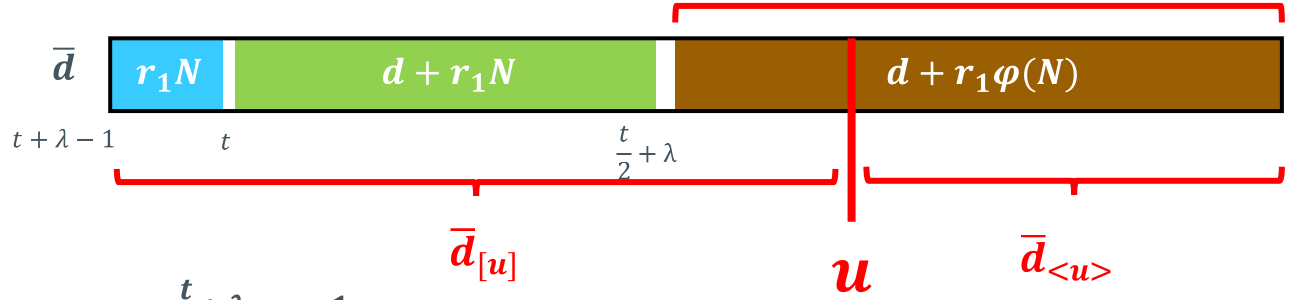
On complete algorithm

- <u>MSB part of d</u>
- Guesses on u bits of d, W bits of H and λ bits of r_1
- → Complete guessed exponent

- Validate guess by checking:

$$\breve{S}_u ?= m^{\overline{d}_{[u]}+\overline{d}_{<u>}} \bmod N$$

On complete algorithm



- <u>LSB part of d</u>
- Let $u \in \left[0, \frac{t}{2} + \lambda\right]$
- As previously, $\bar{d}_{<u>} = \sum_{i=0}^{\frac{t}{2}+\lambda-u-1} 2^i . H_{(i \bmod W)}$
- Approximation of $\bar{d}_{[u]}$:

$$\bar{d}_{[u]} \approx \sum_{i=\frac{t}{2}+\lambda-u}^{t+\lambda-1} 2^i . (d + r_1\varphi(N))_i$$

$$\approx d_{\text{known}} + \sum_{i=\frac{t}{2}+\lambda-v-u}^{\frac{t}{2}+\lambda-v-1} 2^i . \delta_i + \sum_{i=\frac{t}{2}+\lambda-v-u}^{t+\lambda-1} 2^i . (r_1 N)_i + \text{carry}$$

with $\delta_i = (d - (r_1(p + q - 1))_i$

Fault attack

On complete algorithm

- <u>LSB part of d</u>
- Guesses on u bits of d, W bits of H and λ bits of r_1
- → Complete guessed exponent

- Validate guess by checking:

$$\breve{S}_u ?= m^{\overline{d}_{[u]}+\overline{d}_{<u>}} \bmod N$$

- Here, we recover u bits of δ and not of d
- As d and $(p+q-1)$ are fixed values between exponentiations
- We can retrieve d by faulting multiple times at the instant u

On complete algorithm

- Computational complexity:

$$\mathcal{C} = \mathcal{O}\left(\frac{2^{(u+W+\lambda)}.t}{u}\right)$$

- Number of faults:

$$\mathcal{F} = \mathcal{O}\left(\frac{t}{u}\right)$$

- Size of r_2 does not impact the attack, only the size W

- Applicability of the attack depends on the size λ of r_1

Agenda

1. What is a combined attack?
2. Algorithm of Schmidt et al.
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5. Improved algorithm
6. Conclusion

Combined attack

Fault injection and differential side-channel

Algorithm 1 Schmidt *et al.* [20, Alg. 3] left-to-right exponentiation.

Input: $d = (d_{t-1}, \ldots, d_0)_2, m \in \mathbb{Z}_N, N$ and block length W.
Output: $m^d \bmod N$

$r_1 \leftarrow \mathsf{random}(1, 2^\lambda - 1)$
$r_2 \leftarrow \mathsf{random}(1, 2^\lambda - 1)$
$i \leftarrow (r_2^{-1} \bmod N) \cdot r_2$
$R_0 \leftarrow i \cdot 1 \bmod Nr_2$
$R_1 \leftarrow i \cdot m \bmod Nr_2$
$\bar{d} \leftarrow d + r_1 \cdot \varphi(N)$
$[\tilde{d}^{(l-1)}, \ldots, \tilde{d}^{(0)}] \leftarrow [\psi_0(\bar{d}^{(l-1)}), \ldots, \psi_0(\bar{d}^{(0)})]$
$k \leftarrow 0$
$j \leftarrow \mathsf{bitlength}(\tilde{d}) - 1$
while $j \geq 0$ **do**
  $R_0 \leftarrow R_0 \cdot R_k \bmod Nr_2$
  if $(R_0 = 0)$ or $(R_1 = 0)$ **then**
    $[\tilde{d}^{(l-1)}, \ldots, \tilde{d}^{(0)}] \leftarrow [1, \ldots, 1]$
  end if
  $\hat{d} \leftarrow \psi^{-1}_{(R_0 + R_1 \bmod r_2)}(\tilde{d}^{(\lfloor j/W \rfloor)})$
  $k \leftarrow k \oplus \mathsf{bit}(\hat{d}, j \bmod W)$
  $j \leftarrow j - \neg k$
end while
$c \leftarrow R_0 \bmod N$
return c

Fault

→ Remove the exponent blinding:

- bypass (NOP) the call to this function

or

- bypass the multiplication by r_1

Fault injection and differential side-channel

- **Execute the calculation many times (k) on the attacked device**
  - **At each execution i**
    - Fault the step 6. execution
    - Acquire and store the side-channel trace C_i of the exponentiation
  - **Apply with these k curves the differential analysis from Amiel et al.**
    - *Distinguishing multiplications from squaring operations* – SAC 2008.
  - **Allow to recover the secret exponent d**

- ***Attack success depends essentially on the feasibility of the fault injection on the attacked hardware***

Combined attack

Fault injection and template analysis

- **Template pre-processing phase required on the attack device**
  - **Need to store many curves of**
    - The squaring operation $R_0 \times R_0$ with random values R_0
    - The multiplication operation $R_0 \times R_1$ with random values R_0 and R_1
- **Execute the calculation many times (u) on the attacked device**
  - **At each execution i**
    - Fault the step 6. execution
    - Acquire and store the side-channel trace C_i of the exponentiation
  - **Apply with these u curves the Template analysis from Hanley et al.**
    - *Using templates to distinguish multiplications from squaring operations -* International Journal of Information Security, 10. 2011.
  - **Allows to recover the secret exponent d**
- ***Attack success depends essentially on the feasibility of the fault injection on the attacked hardware***

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Improved algorithm

Algorithm 2 Improved Schmidt *et al.* left-to-right exponentiation.

Input: $d = (d_{t-1}, \ldots, d_0)_2, m \in \mathbb{Z}_N, N$ and block length W.
Output: $m^d \bmod N$

$r_1 \leftarrow \mathsf{random}(1, 2^\lambda - 1)$
$r_2 \leftarrow \mathsf{random}(1, 2^\lambda - 1)$
$i \leftarrow (r_2^{-1} \bmod N) \cdot r_2$
$R_0 \leftarrow i \cdot 1 \bmod N r_2$
$R_1 \leftarrow i \cdot m \bmod N r_2$
$\bar{d} \leftarrow d + r_1 \cdot \varphi(N)$ (optional)
$[\tilde{d}^{(l-1)}, \ldots, \tilde{d}^{(0)}] \leftarrow [\psi_0(\bar{d}^{(l-1)}), \ldots, \psi_0(\bar{d}^{(0)})]$
for $i = 0$ **to** $l - 1$ **do**
  $w_i \leftarrow \mathsf{random}(1, 2^W - 1)$
end for
$k \leftarrow 0$
$j \leftarrow \mathsf{bitlength}(\tilde{d}) - 1$
while $j \geq 0$ **do**
  $r_3 \leftarrow \mathsf{random}(1, 2^\lambda - 1)$
  $R_2 \leftarrow R_k + r_3 \cdot N \bmod N r_2$
  $R_0 \leftarrow R_0 \cdot R_2 \bmod N r_2$
  if $(R_0 = 0)$ **or** $(R_1 = 0)$ **then**
    $[\tilde{d}^{(l-1)}, \ldots, \tilde{d}^{(0)}] \leftarrow [w_{l-1}, \ldots, w_0]$
  end if
  $\hat{d} \leftarrow \psi^{-1}_{(R_0 + R_1 \bmod r_2)}(\tilde{d}^{(\lfloor j/W \rfloor)})$
  $k \leftarrow k \oplus \mathsf{bit}(\hat{d}, j \bmod W)$
  $j \leftarrow j - \neg k$
end while
$c \leftarrow R_0 \bmod N$
return c

Replace constant 1 by different w-bit random values for infective operation on exponent

Improved algorithm

Algorithm 2 Improved Schmidt *et al.* left-to-right exponentiation.

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  $R_0 \leftarrow R_0 \cdot R_2 \bmod Nr_2$
  if $(R_0 = 0)$ **or** $(R_1 = 0)$ **then**
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  $\hat{d} \leftarrow \psi^{-1}_{(R_0 + R_1 \bmod r_2)}(\tilde{d}^{(\lfloor j/W \rfloor)})$
  $k \leftarrow k \oplus \mathsf{bit}(\hat{d}, j \bmod W)$
  $j \leftarrow j - \neg k$
end while
$c \leftarrow R_0 \bmod N$
return c

Replace constant 1 by different w-bit random values for infective operation on exponent

Replace squaring operation by multiplications:

$$R_0 \times R_0 \rightarrow R_0 \times (R_0 + Nr_3) \bmod Nr_2$$

→ Combined attacks cannot apply anymore.

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Conclusion

We have presented two new attacks:

- First: a simple fault injection technique
  - Apply with and without the exponent blinding countermeasure
  - Allow to recover the secret exponent with few faulty ciphertexts
- Second: combined attacks
  - Fault injection and Amiel et al. differential analysis
  - Fault injection and Hanley et al. template analysis
- We have presented an improved version of the Schmidt et al. algorithm that thwarts those attacks.

Thanks for your attention …